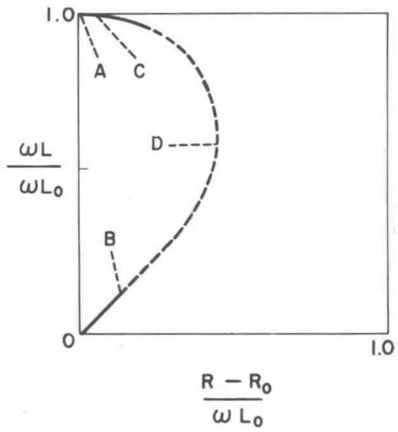
Fig.1 Relative electrical impedance of a coil surrounding a conductive core. Both magnetic permeability and filling factor are unity. Skin depth is infinite at top of curve and zero at bottom



impedance curve. This curve is shown in Fig.1, and it is for unity filling factor and unity permeability. Note that skin depth decreases clockwise in the curve. The end points were obtained by the asymptotic expansions of the Bessel functions (2). The curve has a horizontal tangent at the top, and a 45-deg tangent at the bottom.

The impedance of a coil around a conductive core has been used for a number of years as a gage of the quality of mass-produced commercial items, such as rods, tubes, or bearing balls. Impedance curves similar to Fig.1 have been calculated for many different combinations of filling factor, coil and sample geometry, and sample permeability (3). These curves can be helpful, but it should be noted that our concept of volume measurement is quite different from the commercial method. For product tests, sample volume changes appear as changes in filling factor. In the present technique, volume changes appear as changes in initial inductance. The filling factor does not vary significantly in high-pressure coils.

We note several regions in the curve, indicated by A, B, C, D.

In region A, the skin depth is very large. Small variations in resistivity will have negligible effects on the ac impedance of the coil, but the initial reactance of the coil, L_o, will reflect coil dimensions. There is no simple and exact analytic formula for the inductance of solenoidal coils as a function of dimensions. For single-layer solenoids, however, the relation below is sufficiently accurate in the present application (4).

$$L_o = n^2 r^2 / (9r + 10\ell) \qquad (3)$$

Here, r is coil radius and ℓ is length, both in inches. The number of turns is n. The inductance is given in microhenries. If the coil collapses homogeneously with the sample core, the inductance will be a linear function of any dimension of the coil. For example:

$$\Delta L_o / L_o = \Delta r / r \qquad (4)$$

This type of collapse will be called volume-rule collapse.

In other coil and sample configurations, the coil may collapse so that its cross-sectional area follows the cross-section of the sample. After differentiation of relation (3) and simplification, we obtain

$$\Delta L_o / L_o = (\Delta r / r)(9 + 20 \tfrac{\ell}{r})(9 + 10 \tfrac{\ell}{r})^{-1} \qquad (5)$$

This type of collapse will be called area-rule collapse. The factor of $\Delta r/r$ will be called the area-rule factor. Since this factor varies from 1 to 2, the area-rule collapse yields a somewhat more sensitive coil. Both area and volume-rule collapse are subject to a simple geometrical correction for filling factor.

In region A, sample volume may be determined without any consideration of resistivity effects. It is the preferred region for volume studies. Operation in region A is achieved by selection of very low frequencies in order to insure large skin depth, or by use of a pulverized sample with an insulating binder. Of course, with insulators, one may use higher frequencies, since the skin depth will be large in any event.

In region B, the skin depth is very small. It is seen in the diagram that a change in skin depth will produce equal changes in resistance and reactance. This may be expressed analytically as follows:

$$\omega L \sim \omega L_o \, \rho^{\frac{1}{2}} \qquad (6)$$

$$R - R_o \sim \omega L_o \, \rho^{\frac{1}{2}} \qquad (7)$$

Only one parameter could be observed, the product

$\omega L_o \rho^{\frac{1}{2}}$, and so the volume and resistivity data could not be separated.

The situation may be improved by a filling factor which is not unity, perhaps about 0.6. It may be shown both theoretically and experimentally that this does not change the resistance of the coil appreciably, but the inductance is greatly increased by the volume of space available for the magnetic flux. The inductance of the coil is proportional to the sum of two volumes of space: the effective small volume in the skin-depth region of the core, and the larger volume outside the core. The contribution to coil inductance by the skin-depth volume is governed by equation (6). The contribution to coil inductance by the external volume is proportional to the extent of the magnetic field; i.e., to coil size. This effect has been discussed in the testing handbook mentioned earlier (3). The following empirical relations are readily derived upon consideration of equations (6) and (7) and the foregoing discussion:

$$\omega L = \omega L_o \,(km\rho^{\frac{1}{2}} + b) \qquad (8)$$

$$R - R_o = \omega L_o m\rho^{\frac{1}{2}} \qquad (9)$$

the constant k is inserted because the slope of the impedance curve is not exactly unity for a finite frequency of operation. The constants b and m are determined by comparison of coil impedance with and without the core. The constant k is calculated directly from the Bessel functions. The desired terms ρ and L_o are then easily computed from the observed data. The addition of inactive volume has in effect given us an additional piece of information, proportional solely to the initial inductance. Volume and resistivity may then be separated.

Region B is preferred for study of resistivity changes. The great virtue here is that a low-resistivity sample may be studied from a relatively high ac resistance. Furthermore, large resistivity changes often swamp volume changes in the information-mixing process and permit immediate interpretation of data.

In region C, the skin depth and sample dimensions are comparable. The resistance of the coil is dependent on the resistivity of the core, and the inductance is proportional to the volume. This is the region to which first reference was made in the introduction. This region is especially useful for preliminary surveys in which qualitative data is sufficient. The data can, if desired, be analyzed by calculation.

In region D, a change in resistivity produces a change in the inductance only, the same as a change in volume. The results in this region are completely indeterminate.

The discussion so far has been devoted to samples of unit permeability. In ferromagnetic samples, the variation of permeability with pressure introduces a third variable. An additional observational parameter is needed to characterize the sample. This may be obtained from a second coil wrapped over the first. The observed parameters are resistance and reactance of the first coil, and voltage induced in the second coil. This voltage will depend on the mutual coupling between coils, and this coupling is determined in large part by the core permeability. We have not yet engaged in a thorough study of this particular measurement; the theory has been treated in the testing handbook (3).

EXPERIMENTAL APPARATUS

Electrical Considerations

Test Coils. It is usually desirable to have the coil filling factor near unity, except in case of measurements in region B. A high filling factor increases the sensitivity, and insures that the coil collapse will more faithfully follow the sample. In region B, a filling factor of about 60 percent is suitable.

The test coil would seem to be subject of unlimited variation in design. It is tempting to optimize sensitivity by juggling coil parameters. We have found, however, by trying a number of different sizes and configurations, that a coil of initial inductance 2 to 8 μh is suitable for all measurements from 1000 cps to 10 mc. The coil inductance will then exceed the inductance of eight feet of coaxial lead by a factor of 3 or 4 at 1000 cps. At 2 mc, the ac resistance of the coil will be several hundred ohms with the usual metallic samples, and the data with eight feet of coaxial cable yield resistivity to good approximation even without the use of a Smith chart.

Two standardized coil designs were adopted. Our large standard coil consisted of 25 turns of 35-gage copper wire spaced on a silver chloride form $\frac{1}{2}$ in. long by 5/16 in. dia. The enclosed sample core was $\frac{1}{2}$ in. long by $\frac{1}{4}$ in. dia. This coil had an inductance of 2.8 μh. The small standard coil had 25 turns of 38-gage wire closely wound directly on the central portion of the core, which was $\frac{1}{4}$ in. long by $\frac{1}{4}$ in. dia. This coil had an inductance of 4.2 μh.

The coil was connected to the coaxial cable by two lengths of 14-gage copper wire about four inches long.

Instrumentation. Low-frequency measurements were made with an inductance bridge (6). High